EFFECT OF INITIAL STRESSES ON THE DEVELOPMENT OF A BRITTLE CRACK

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The importance of initial tensile stresses (residual stresses) as a factor facilitating brittle fracture was first noted some time ago. It is obvious that the initial stress acting together with the stress due to the external load can considerably reduce the external breaking force. However, the role played by the initial stresses in brittle fracture is not restricted to this fact. The dangerous thing about initial stresses is that they can create conditions facilitating unstable development of a crack produced in their zone of action. For this reason residual stresses near a welded joint can produce spontaneous brittle fracture in unfavorable conditions (low ambient temperature, presence of a crack, random external loads, etc.). Cases of sudden failure of ship hulls, large tanks, and other welded structures [1, 2] have been reported at relatively low applied stresses as a result of the sudden development of a crack. Propagation of the crack began near a weld. Several experimental investigations [3, 4] concerned with these failures have dealt with the effect of residual stresses on the breaking load in brittle fracture. We have assessed the effect of initial stresses (and, in particular, welding stresses) on the stability of a crack existing in the initial stress zone.

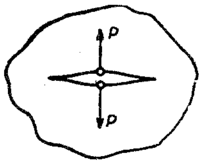

Fig. 1

1. Constant initial stresses. Let us assume that the field of initial tensile stresses contains a linear crack. The following relation [5] determines the critical length $2l^*$ of this crack:

$$l^* = \frac{2K^2}{\pi^2 Q^2} .$$

Here K is the modulus of cohesion of the material, and Q is the intensity of the initial tensile field of the initial stresses along the crack axis.

Initial cracks with a half-length $l_0 < l^*$ are in stationary equilibrium and, consequently, stable. Cracks with a half-length close to the critical length are most dangerous.

It is obvious that a field of compressive initial stresses may contain closed cracks of any length.

The behavior of the cracks changes if an additional variable stress field due to an external load increasing, for example, in proportion to a certain parameter λ, is created in the crack zone.

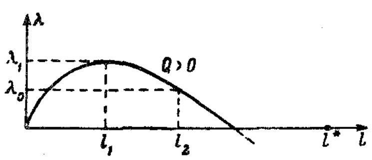

Fig. 2

Let us assume that the external load is such that without an initial stress field the crack would be stable. For the sake of definiteness, we will describe a case in which two balanced forces, tending to open the crack, are applied (Fig. 1). The relationship between the parameter λ and l is in this case [5]

$$\lambda_0(l) = K\sqrt{2l} .$$

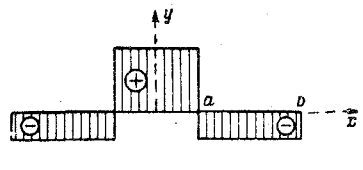

Fig. 3

The existence of an initial field of constant stresses Q is taken into account by the function

$$r(l) = -\frac{\pi Q}{2K}\sqrt{2l} ,$$

which is independent of the nature of the variable external load [6]. Then

$$\lambda(l) = \lambda_0(l)\,[1 + r(l)] = K\sqrt{2l}\left(1 - \frac{\pi Q}{2K}\sqrt{2l}\right).$$

Figure 2 shows the relationship between λ and l for this case. If the half-length of the initial crack is

$$l_2 = l_0 > l_1 = \frac{K^2}{2Q^2\pi^2} = \frac{1}{4} l^*$$

then the crack opens completely at $\lambda = \lambda_0$ and becomes unstable if the parameter λ continues to increase. If, in particular, the length of the initial crack $2l_0 = 2l_1$, then the critical force is

$$P_1 = \lambda_1 = K^2 / 2\pi Q ,$$

i.e., the critical load decreases sharply with increasing intensity of the initial stress field.

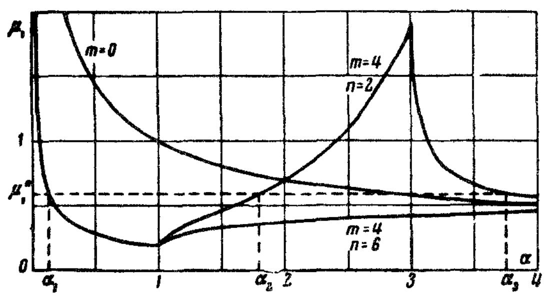

Fig. 4

If the length of the initial crack $2l_0 < 2l_1$, then an increase of λ initiates stable development of the crack. This development continues until the crack length becomes $2l_1$ (for $\lambda = \lambda_1$). Thereupon the crack becomes unstable. A further increase of the parameter λ initiates rapid propagation of the crack and its length may exceed $2l^*$. In this case removal of the external load will not stop the propagation of the crack: its development will be ensured by the initial stress field alone.

2. Effect of welding stresses on the development of a crack. In Fig. 3, in order to assess the effect of welded joints on the development of a crack, we show a diagram of welding stresses. Tensile stresses Q operate in the central region along the x axis, and compressive stresses T = Q/n on either side. In addition, we assume that the same region also contains a uniform tensile stress field $Q_0 = Q/m$ (n and m are real numbers). In this case the normal stresses $p_0(x)$ operating along the axis of symmetry x are

$$p_0(x) = \begin{cases} (m+1)Q_0 & (0 \leqslant x \leqslant a) \\ -\frac{m-n}{n} Q_0 & (a \leqslant x \leqslant b) \\ Q & (b \leqslant x < \infty). \end{cases}$$

We now assume that at the origin of the coordinate system there is a crack (slot) of length $2l_0$ pointing in the direction of the x axis.

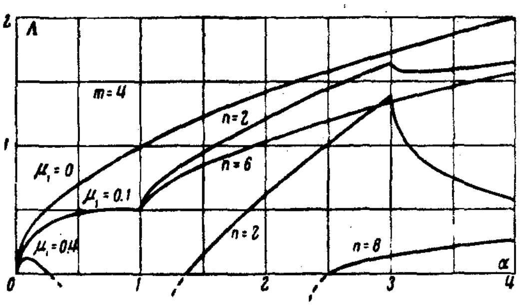

Fig. 5

The relationship between the length of the equilibrium crack $2l$, the intensity of the stress field Q_0, and the parameters m and n is expressed by the following relations

$$\mu_1 = \frac{1}{(m+1)\sqrt{\alpha}} \quad (\alpha \leqslant 1) \quad \left(\alpha = \frac{l}{a}\right),$$

$$\mu_1 = \frac{\pi}{2\sqrt{\alpha} f_2(\alpha)} \quad (1 \leqslant \alpha \leqslant n+1) \quad \left(\mu_1 = \frac{\pi \sqrt{a}}{K\sqrt{2}} Q_0\right),$$

$$\mu_1 = \frac{\pi}{2\sqrt{\alpha} f_3(\alpha)} \quad (n+1 \leqslant \alpha < \infty),$$

$$f_2(\alpha) = \frac{m(n+1)}{n} \arcsin\frac{1}{\alpha} - \frac{m-n}{n}\frac{\pi}{2},$$

$$f_3(\alpha) = \frac{m(n+1)}{n} \arcsin\frac{1}{\alpha} - \frac{m}{n} \arcsin\frac{n+1}{\alpha} + \frac{\pi}{2}.$$

Here it is taken into account that $Qa = T(b - a)$.

Figure 4 shows the relationship between Q_0 and α for the cases m = 4, n = 2, 4, and 6. The same diagram contains the relationship between Q_0 and α without the welding stress field (m = 0).

The graphs show, in particular, that at a certain Q_0 ($\mu_1 = \mu_1^*$) the welding stress field (e.g., m = 4, n = 2) may contain initial cracks with half-lengths $l_0 < l_1$ and $l_2 < l_0 < l_3$; it may also contain moving stable cracks.

We now assume that the region which contains the cracks also contains, in addition to the fixed stress fields already considered, an increasing stress field of intensity $\lambda g(x)$ along the axis of symmetry. As in §1, we again assume that without the initial stresses the variable external load would cause stable development of the crack.

For the sake of definiteness, we again assume that

$$\lambda_0(l) = K\sqrt{2l}.$$

Using the known equations [5, 6], we obtain the following basic relations

$$\Lambda = \sqrt{\alpha} - \mu_1(m+1)\alpha \quad (0 \leqslant \alpha \leqslant 1),$$

$$\Lambda = \sqrt{\alpha} - 2\pi^{-1} f_2(\alpha)\mu_1\alpha \quad (1 \leqslant \alpha \leqslant n+1) \quad \left(\Lambda = \frac{\lambda}{K\sqrt{2a}}\right),$$

$$\Lambda = \sqrt{\alpha} - 2\pi^{-1}\mu_1 f_3(\alpha)\alpha \quad (n+1 \leqslant \alpha < \infty).$$

Figure 5 shows curves representing the propagation of a crack with increasing parameter λ for various internal stress fields.

For example, the graph shows that at $\mu_1 = 0.4$ an initial crack of length $\alpha \simeq 3.5$ (according to Fig 4 a crack of this length can exist in the given initial stress field (m = 4, n = 2)) begins spontaneous propagation when the parameter λ reaches the corresponding value λ^*. On the other hand, without an initial stress field ($\mu_1 = 0$) an increase of λ causes a crack of this length to grow, but stably and only when λ is considerably larger than λ^*.

If the length of the initial crack $l_2 < l_0 < 3a$, then an increase of λ initiates stable growth of the crack.

It should be noted that at $\mu_1 = 2.0$ initial cracks of length $\alpha = 0.0025$ begin to spread catastrophically from $\lambda = 0.25 K\sqrt{2a}$ onwards (Fig. 6). However, cracks of length $\alpha > 0.0025$ (on reaching the corresponding value of λ) also become unstable.

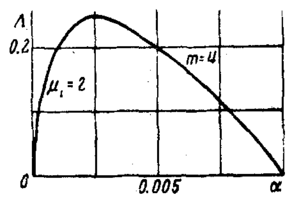

Fig. 6

This brief analysis illustrates the fact that varying, generally speaking, "safe" loads can cause catastrophic failure given a certain combination of initial welding stresses.

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